Spring 2015 **Quiz 1** **Date: February 20**
K.Yaghi **Math 201- Section X** **Duration: 1 hour**

Problem 1 *(answer on page* 1 *of the booklet)*
Which of the following sequences converge, and which diverge? Find the limit of each convergent sequence. (7 *pts each*)

a) $a_n = 3^{1/n}\cos(n!)\sin(\frac{1}{n})$ b) $b_n = (\frac{n+ln3}{n-ln2})^n(\frac{n^2-n}{2n^2+2n-6})$ c) $c_n = (n+1)\tan(\frac{1}{n})$

Problem 2 (*answer on pages* 2 *&* 3 *of the booklet*)
Which of the following series converge, and which diverge? When possible find the sum of the series. (8 *pts each*)

a) $\sum_{n=2}^{\infty}\frac{2^n}{4^{n+1}}+\frac{(-1)^{n+1}3^{n-2}}{7^n}$ b) $\sum_{n=1}^{\infty}\frac{n^2+2}{e^{2n}(n+1)^2}$ c) $\sum_{n=2}^{\infty}\frac{(-1)^n}{n\ln n}$ d) $\sum_{n=2}^{\infty}(1-\cos(\frac{1}{n^2}))^{1.2}$

Problem 3 (*answer on page* 4 *of the booklet*)
Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty}\frac{(-1)^n(\ln n)^2(x-5)^n}{2^n\, n^{1.4}}$$

For what values of x does the series converge absolutely? Conditionally? (20 *pts*)

Problem 4 (*answer on pages* 5, 6 *& the last page of the booklet*)

a) (5 *pts*) Write a power series expansion for the function $f(x) = \sin x$ about the point $x = 0$. Also find the taylor polynomials $p1(x)$ and $p3(x)$ generated by $f(x)$ about the point $x = 0$.

b) (6 *pts*) Use the alternating series estimation theorem to estimate the error resulting from the approximation
$$\sin\left(\frac{\pi}{6}\right) \approx p3(?)$$
Does $p3(?)$ tend to be too small or too large?

c) (7 *pts*) Use taylor's theorem to prove that
$$\left|\sin x - x + \frac{x^3}{6}\right| \leq \frac{|x|^5}{120}$$

d) (5 *pts*) Decide if $\sum_{n=2}^{\infty}\sqrt{\frac{1}{n}-\sin(\frac{1}{n})}$ converge or diverge? Justify your answer.

e) (4 *pts*) Find the following sum

$$\sum_{n=1}^{\infty}\frac{(-1)^n(\frac{\pi}{3})^{2n+1}+(-1)^n(2n+1)(\frac{\pi}{3})^{2n}}{(2n+1)!}$$

Good Luck & Best Wishes

K. Yaghi